

MIMO Channel Estimation Using the LS and MMSE Algorithm

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Abstract: Wireless Communication Technology has developed over the past few years for other objectives. The Multiple Input Multiple Output (MIMO) is one of techniques that is used to enhance the data rates, in which multiple antennas are employed both the transmitter and receiver. Multiple signals are transmitted from different antennas at the transmitter using the same frequency and separated space. Various channel estimation techniques are employed in order to judge the physical effects of the medium present. In this paper, we analyze and implement various estimation techniques for MIMO Systems such as Least Squares (LS), Minimum Mean Square Error (MMSE), these techniques are therefore compared to effectively estimate the channel in MIMO System. The results demonstrate that SNR required to support different values of bit error rate varies depending on different low correlation between the transmitting and the receiving antennas. In addition, it is illustrated that when the number of transmitter and receiver antennas increases, the performance of TBCE schemes significantly improves. The same behavior is also observed for MIMO system. Performance of both MMSE and LS estimation are the same for all kinds of modulation at small value of SNR but the more we increase the SNR value the more performance gap goes on increasing.

Keywords: Channel estimation, Minimum mean square error (MMSE), Least square (LS), Kalman filter, Orthogonal frequency division multiplexing.

I. Introduction

In recent years, Multi-Input Multi-Output (MIMO) communications are introduced as an emerging technology to offer significant promise for high data rates and mobility required by the next generation wireless communication systems. Using multiple transmit as well as receive antennas, a MIMO system exploits spatial diversity, higher data rate, greater coverage and improved link robustness without increasing total transmission power or bandwidth. However, MIMO relies upon the knowledge of Channel State Information (CSI) at the receiver for data detection and decoding [1] [2]. It has been proved that when the channel is flat fading and perfectly known to the receiver, the performance of a MIMO system grows linearly with the number of transmit or receive antennas, whichever less is [3]. Therefore, an accurate and robust channel estimation is of crucial importance for coherent demodulation in wireless MIMO systems [7].

Use of MIMO channels, when bandwidth is limited, has much higher spectral efficiency versus Single-Input Single-Output (SISO), Single-Input Multi-Output (SIMO), and Multi Input Single-Output (MISO) channels [4]. It is shown that the maximum achievable diversity gain of MIMO channels is the product of the number of transmitter and receiver antennas. Therefore, by employing MIMO channels not only the mobility of wireless communications can be increased, but also its robustness [6]. Mobile communication systems transmit information by changing the amplitude or phase of radio waves. In the receiving side of mobile system, amplitude or phase can vary widely [11]. This causes degradation in the quality of system since the performance of receiver is highly dependent on the accuracy of estimated instantaneous channel. However, these detectors require knowledge on the channel impulse response (CIR), which can be provided by a separate channel estimator to minimize the error probability [5].

In this paper, we analyze a LS and MMSE channel estimators, in MIMO system, that signal detector needs to know channel impulse response (CIR) characteristics to ensure minimum Mean Square Error of the Channel Estimation and to minimize the error probability. We present an efficient MIMO channel estimation with training sequences. The proposed algorithm has been designed and simulated using Matlab Program then tested and evaluated the efficiency of the proposed algorithms.

Outlines The rest of this paper is organized as follows. We present the system description Sect. 2. Section 3 introduces the basics of a phrase-based SMT system, which is regarded as the basis of our experiments. Section 4 will illustrate the feature factors (POS and CCG) that parse the tag and supertag of the corpus. In Sect. 5, we will explain how n-gram LM was included in the translation process. Section 6 will introduce the basis of factored translation model and show its similarity to phrase-based SMT models. In Sect. 7, we will explore our experiments and results on the four models with BLEU scores in the presence of various high n-gram language models. Section 8 will comprehensively conclude the work.

II. System Description

Consider a MIMO system equipped with N_T transmit antennas and N_R receive antennas. The block diagram of a typical MIMO 2x2 is shown in Figure. 1.

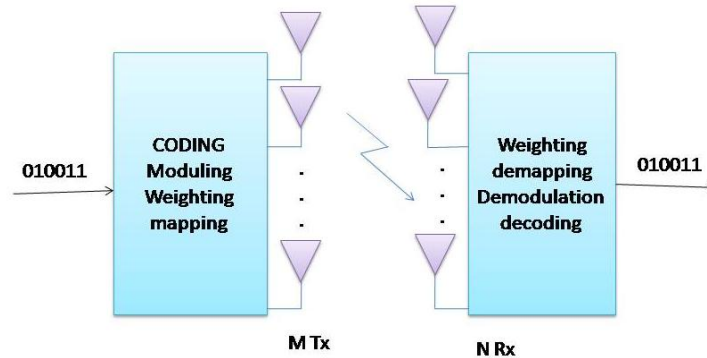


Figure.1 General Architecture of a MIMO

It is assumed that the channel coherence bandwidth is larger than the transmitted signal bandwidth so that the channel can be considered as narrowband or flat fading [10]. Furthermore, the channel is assumed to be stationary during the communication process of a block. Hence, by assuming the block Rayleigh fading model for flat MIMO channels, the channel response is fixed within one block and changes from one block to another one randomly. During the training period, the received signal in such a system can be written as (1)

$$Y = H \cdot S + N \tag{1}$$

where Y , S and N are the complex N_R -vector of received signals on the N_R receive antennas, the possibly complex N_T -vector of transmitted signals on the N_T transmit antennas, and the complex N_R -vector of additive receiver noise, respectively.

The elements of the noise matrix are independent and identically distributed complex Gaussian random variables with zero-mean and σ_n^2 variance, and the correlation matrix of N is then given by [8]:

$$R_{nn} = E\{N^H \cdot N\} = \sigma_n^2 \cdot N_R \cdot I_{N_R} \tag{2}$$

where $(\cdot)^H$ is reserved for the matrix hermitian, $E(\cdot)$ is the mathematical expectation, and I_{N_R} denotes the $N_R \times N_R$ identity matrix. N_p the number of transmitted training symbols by each transmitter antenna. The matrix H in the model (3.1) is the $N_R \times N_T$ matrix of complex fading coefficients. The (m,n) -th element of the matrix H denoted by $h_{m,n}$ represents the fading coefficient value between the m -th receiver antenna and the n -th transmitter antenna. Here, it is assumed that the MIMO system has equal transmit and receive antennas. The elements of H and noise are independent of each other. In order to estimate the channel matrix, it is required that $N_p \geq N_T$ training symbols are transmitted by each transmitter antenna. The function of a channel estimation algorithm is to recover the channel matrix H based on the knowledge of Y and S . Output (received) signals in locations $Canare$ as follow:

$$\begin{aligned} y_{n1} &= h_{11} \cdot s_1 + h_{21} \cdot s_2 + n_1 \\ y_{n2} &= h_{12} \cdot s_1 + h_{22} \cdot s_2 + n_2 \end{aligned}$$

2.1 Signal Model:

We consider a flat fading MIMO wireless system with N_T transmit and N_R receive antennas. The symbol transmitted by antenna m at time instant k is denoted by $S_m(k)$. The transmitted symbols are arranged in the vector

$$S_M(k) = [s_1(k), \dots, s_{N_T}(k)]^T \tag{3}$$

of length N_T , where $(\cdot)^T$ denotes the transpose operation. Between every transmit antenna m and every receive antenna n there is a complex single-input single-output (SISO) channel impulse response $h_{n,m}(k)$ of length $L+1$, described by the vector

$$h_{n,m} = [h_{n,m}(0), \dots, h_{n,m}(L)]^T \tag{4}$$

Assuming the same channel order L for all channels, the MIMO channel can be described by $L+1$ complex channel matrices

$$H(k) = \begin{bmatrix} h_{n,m}(k) & \dots & h_{n,m}(k) \\ \vdots & \ddots & \vdots \\ h_{N_R,1}(k) & \dots & h_{N_R,N_T}(k) \end{bmatrix}, k = 0, \dots, L \tag{5}$$

of the dimension $N_R \times N_T$. The symbol received by antenna n at time instant k is denoted

by $Y_n(k)$. The symbols received by the N_R antennas are arranged in a vector

$$Y(k) = [y_1(k), \dots, y_{N_R}(k)]^T \tag{6}$$

of length N_R , which can be expressed with (3.1), (3.3) and $n(k)$ as noise vector of length N_R as

$$\begin{aligned} y_n &= S_1 h_{n,1} + \dots + S_{N_T} + n \\ &= [S_1, \dots, S_{N_T}] \begin{bmatrix} h_{n,1} \\ \vdots \\ h_{n,N_T} \end{bmatrix} + n \end{aligned} \tag{7}$$

We assume additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2 per receive antenna, i.e. the spatial correlation matrix of the noise is given by

$$R_{NN} = E\{n(k)n^H(k)\} = \sigma_n^2 \cdot N_R \cdot I_{N_R} \tag{8}$$

where I_{N_R} is the identity matrix and $(\cdot)^H$ denotes the complex conjugate(Hermitian) transpose. The receive vector y_N follows to

$$y_n = S h_n + n \tag{9}$$

2.2 QPSK Modulator/ Demodulator

PSK is a digital modulation technique which is most commonly used modulation technique in present digital communication systems [12]. In PSK modulation, the phase of the carrier is altered in accordance with the input binary coded information [9]. The PSK is further subdivided into BPSK, 8-PSK, 16-PSK, QPSK, DPSK. In binary phase shifting keying the transmitted signal is sinusoid of fixed amplitude, has fixed phase as shown in Figure.2.

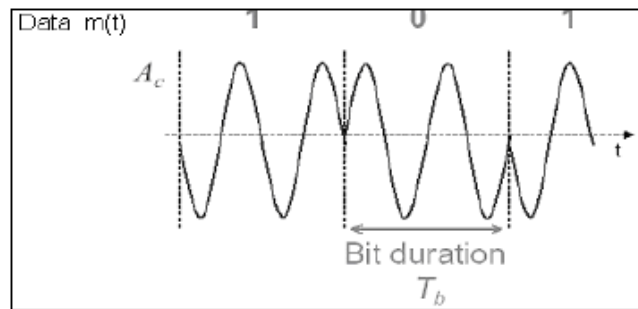


Figure.2 Phase Shift Keying

QPSK [4] is a phase modulation scheme, used in constellation mapping. Here the input bits stream is converted into complex stream using equation 10 and where the I and Q both are in phase with I-out and Q-out respectively. QPSK modulator accepts the binary bits as inputs taken as a symbol and converts them into complex value. QPSK takes only 4 symbols and generate its complex value in this fashion.

$$D = (I + jQ)P * K_{MOD} \tag{10}$$

Where $K_{MOD}=1/1.414$

III. Perfect Channel Estimation

Perfect estimator is the simplest algorithm to estimate the channel matrix. By setting the noise equal to zero in (1), the perfect approach estimates the channel matrix as

$$H_{perfect} = Y \cdot S^{-1} \tag{11}$$

In this way the channel matrix is simply will be obtained by inverse matrix of S/Y .

Least Square Algorithm:

In this case we estimate the free noise MIMO channel perfectly. Perfect estimation will be used as a lower bound. Consider a Rayleigh flat-fading MIMO channel characterized by H , S as the training sequence, Y as related received signal. N represents Additive White Gaussian Noise. If we assume that:

$$Y = SH + N \tag{12}$$

(12)

LS estimator finds H that $SH \approx Y$. [6],[7]LS Algorithm, minimizes the Euclidian distance of $SH - Y$.

Table 3.1 Table 3.1 Show the Input and the Output of QPSK modulator

Input Bits	I-Out	Q-Out
00	-1	-1
01	-1	+1
10	+1	-1

11	+1	+1
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For this minimization we do following steps:

$$\begin{aligned} \|\mathbf{S}\hat{\mathbf{H}} - \mathbf{Y}\|^2 &= (\mathbf{S}\hat{\mathbf{H}} - \mathbf{Y})^H (\mathbf{S}\hat{\mathbf{H}} - \mathbf{Y}) \\ &= (\mathbf{S}\hat{\mathbf{H}})^H (\mathbf{S}\hat{\mathbf{H}}) - \mathbf{Y}^H \mathbf{S}\hat{\mathbf{H}} - (\mathbf{S}\hat{\mathbf{H}})^H \mathbf{Y} + \mathbf{Y}^H \mathbf{Y} \end{aligned} \quad (13)$$

After derivation in respect to $\hat{\mathbf{H}}$ and to put the equation equal the zero:

$$2\mathbf{S}^H \mathbf{S}\hat{\mathbf{H}} - 2\mathbf{S}^H \mathbf{Y} = \mathbf{0} \rightarrow \mathbf{S}^H \mathbf{S}\hat{\mathbf{H}} = \mathbf{S}^H \mathbf{Y} \quad (14)$$

And so we will have:

$$\hat{\mathbf{H}} = (\mathbf{S}^H \mathbf{S})^{-1} \mathbf{S}^H \mathbf{Y} \quad (15)$$

We use formula (3.15), as the LS channel estimation algorithm.

Minimum Mean Square Error Channel Estimation:

The Minimum Mean Square Error (MMSE) channel estimates given by [5]

$$\hat{\mathbf{h}}_{n,MMSE} = [\mathbf{S}^H \mathbf{R}_{nn}^{-1} \mathbf{S} + \mathbf{R}_{hh}^{-1}]^{-1} \mathbf{S}^H \mathbf{R}_{nn}^{-1} \mathbf{Y}_n \quad (16)$$

Where $\mathbf{R}_{hh} = \mathbf{E}\{\mathbf{h}_n \mathbf{h}_n^H\}$

and in the case of additive white noise the MMSE channel estimate follows to

$$\hat{\mathbf{h}}_{n,MMSE} = [\mathbf{S}^H \mathbf{S} + \frac{\sigma_n^2}{\sigma_h^2}]^{-1} \mathbf{S}^H \mathbf{Y}_n \quad (17)$$

Setting the term $\frac{\sigma_n^2}{\sigma_h^2}$ in (3.18) to zero yields the Least Square channel estimate in the case of additive white noise.

Meansquareerrorofthechannelestimation:

The correlation matrix of the error of the channel estimations given by

$$\mathbf{R}_{ee} = \{[\mathbf{h}_n - \hat{\mathbf{h}}_n][\mathbf{h}_n - \hat{\mathbf{h}}_n]^H\} \quad (18)$$

For the Minimum Mean Square Error channel estimation it follows to [5]

$$\mathbf{R}_{ee} = [\mathbf{S}^H \mathbf{R}_{nn}^{-1} \mathbf{S} + \mathbf{R}_{hh}^{-1}] \quad (19)$$

The Mean Square Error (MSE) of a MIMO channel

$$\mathbf{MSE}_{MIMO} = \mathbf{E}\{[\hat{\mathbf{h}}_n - \mathbf{h}_n][\hat{\mathbf{h}}_n - \mathbf{h}_n]^H\} = \sigma_n^2 \cdot N_t \cdot \text{tr}(\mathbf{R}_{ee}) \quad (20)$$

is the trace of the error correlation matrix \mathbf{R}_{ee} . The trace of a matrix denoted by $\text{tr}(\cdot)$ is the sum of the diagonal elements. For additive white noise the Mean Square Error follows to

$$\mathbf{MSE}_{MIMO} = \sigma_n^2 \cdot N_t \cdot \text{tr}([\mathbf{S}^H \mathbf{S} + \frac{\sigma_n^2}{\sigma_h^2} \mathbf{I}]^{-1})$$

(21)

For the Least Square channel estimation the term $\frac{\sigma_n^2}{\sigma_h^2}$ has to be set to zero.

IV. Simulation Results

In this work the Simulation Results of the channel estimation are presented. The chosen network simulate or, c Matlab. The simulation results that are collected from the implementation of both the LS and MMSE using the Matlab simulation are presented.

4.1 Simulation Model And Results

It will be useful to provide a simple Matlab example simulating a QPSK transmission and reception in Flat MIMO channel. The script performs the following

- 1- Generate random binary sequence of 1's and 0's.
- 2- Mapping the binary sequence using QPSK.
- 3- Multiply the symbols with the channel and then add white Gaussian noise.
- 4- At the receiver, equalize (divide) the received symbols with the known channel.
- 5- Perform hard decision demapping and count the bit errors.
- 6- Repeat for multiple values of SNR and plot the simulation

Here, simulation results and derived performance metrics of before mentioned algorithms will be explained. flat fading MIMO channel used for training-based estimating channel After , random data is generated in the transmitter and modulated signal will be sent through the channel. By counting number of errors, BER will be extracted. In the perfect channel estimation, we haven't used the AWGN in estimating process, but it is used in calculating the error. 8 bits training sequence for a MIMO 2x2 system has been considered. We also used a QPSk modulator for modulation data in transmitter. In addition, 100 iterations for calculating BER and MSE which each contains 400 bits have been used. In this section, we evaluate the BER to estimate the channel conditions. Figure 3 shows BER comparison of MMSE and LS with respect perfect channel. Here we consider MIMO based system for high multimedia communication system. The parameter

consider for simulation is (number of transmitter) $T_x=2$, (number of receiver) $R_x = 2$, .5 correlation coefficient and 8 bits training sequence . Fig 3shows the BER comparison between LS and MMSE from which it is clear that MMSE is better technique than LS which does not utilize the channel statistics. At high SNR values, the performance gap is more than at low SNR. But for improved performance in MMSE we have to pay for more complexity which results in increased computational time and high implementation cost of hardware to have a priori knowledge of channel behavior.

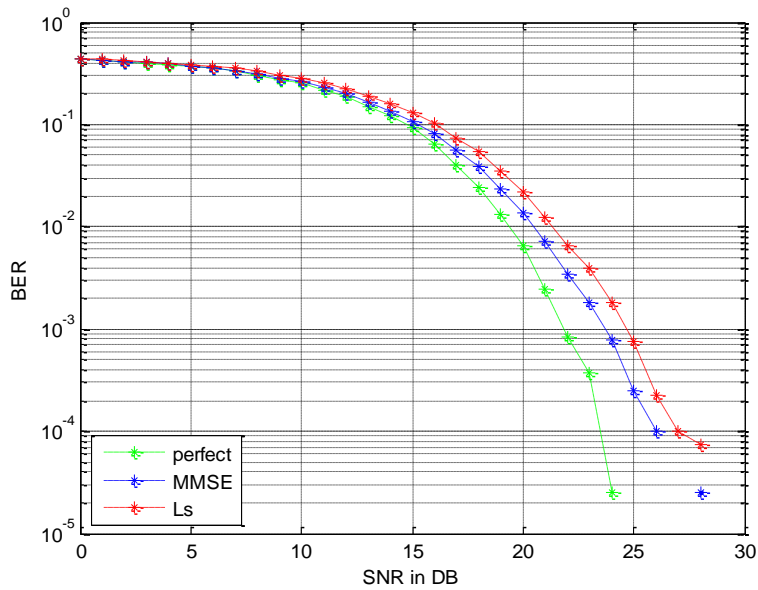


Figure 3 comparison of MMSE with LS with respect to perfect channel

Fig. 4 illustrates the MSE comparison of MMSE with LS estimators versus different SNR for flat fading MIMO 2×2 channel. It is obvious that, increasing SNR is the reason for decreasing MSE.

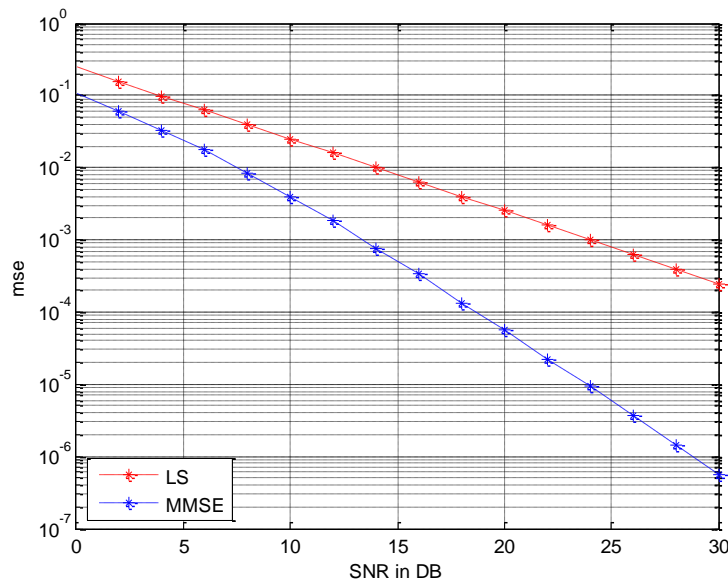


Figure 4 MSE comparison of MMSE with LS

The parameter consider for simulation is (number of transmitter) $T_x=2$ and (number of receiver) $R_x=2,3,4$,.5 correlation coefficient , 8 bits training sequence and QPSK modulation technique for data transmission. Figure 5 and 6 show comparison of MMSE and LS with different antennas values, increasing the number of transmit antennas leads to increase the performance estimators, but it is highlighted in LS. For low SNRs, this approximation effect is small compared to the channel noise, while it becomes dominant for large SNRs. The curves level out to a value determined in the energy of the taps. MMSE estimator reduces the

mean square error for a range of SNRs compared to LS estimator. As before, increasing the SNR is the reason for decreasing BER of all estimators but it is more effective for LS one.

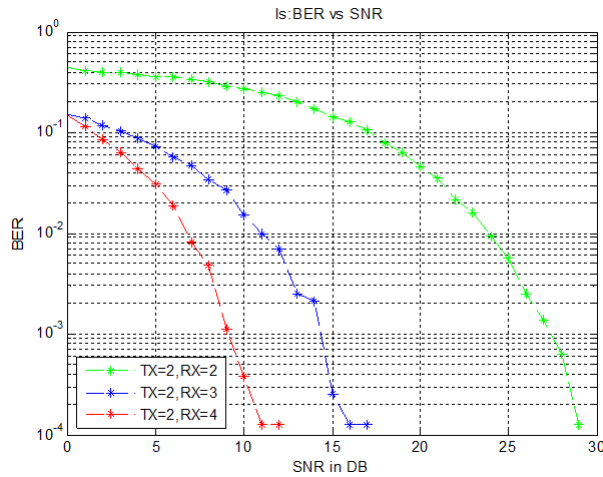


Figure 5 BER comparison LS with respect to different antennas

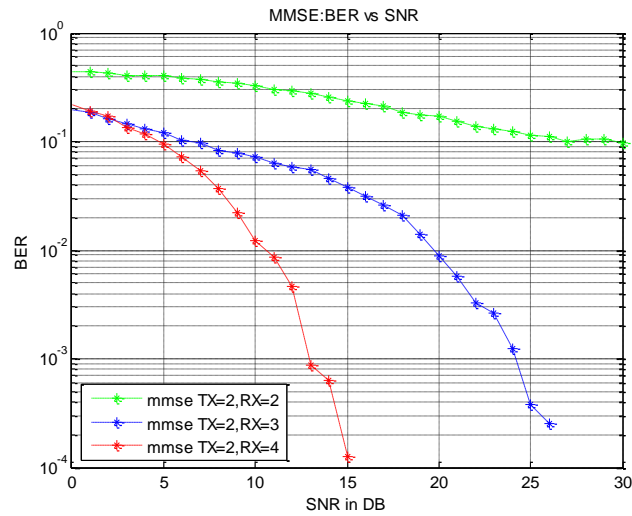


Figure 6 BER comparison MMSE with respect to different antennas

The parameter consider for simulation is (number of transmitter) $T_x=2$ and (number of receiver) $R_x=2$, correlation coefficients of .4 , .5 and .6 , 8 bits training sequence and QPSK modulation technique for data transmission.

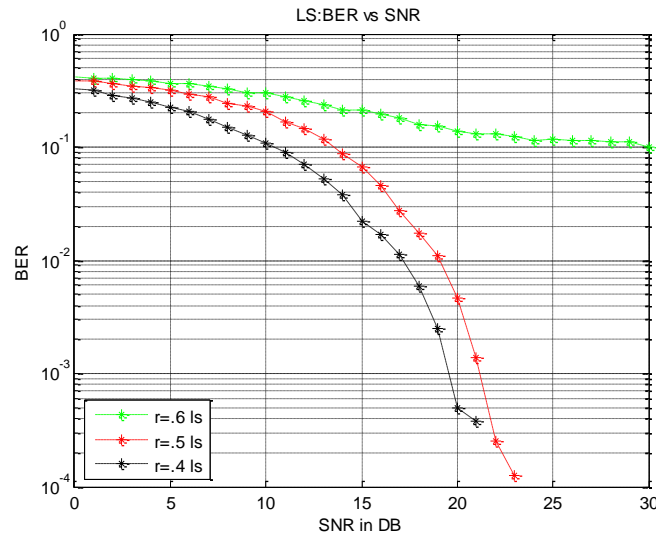


Figure 7 BER comparison LS with respect to correlation coefficients

Figure 7 and 8 show BER comparison of MMSE and LS with different correlation for 2x2 MIMO systems using flat fading channel with, that BER decreases with low correlation values. The results demonstrate that SNR required to support different values of bit error rate varies depending on different low correlation between the transmitting and the receiving antennas. The parameter consider for simulation is (number of transmitter) $T_x=2$ and (number of receiver) $R_x=2$, .5 correlation coefficients, 8 bits training sequence and BPSK, QPSK, 8-PSK and 16-PSK modulation technique for data transmission.

Figure 9 and 10 demonstrate the performance of MIMO system employing different modulation techniques for data transmission. As from the general modulation theory we know the performance is better for less order modulation techniques as compared to high order modulation but in high order modulation we have larger data rate. Same behavior is also observed for MIMO system. Performance is same for all kinds of modulation at small value of SNR but as we increase the SNR value the performance gap goes on increasing. Outperforms all other modulations techniques. At low SNR, the performance is same for all modulations but the increasing effect in SNR has clear demonstration of the difference of performance. So for high SNR, we choose modulation according to the system requirement but for low SNR we can choose any one.

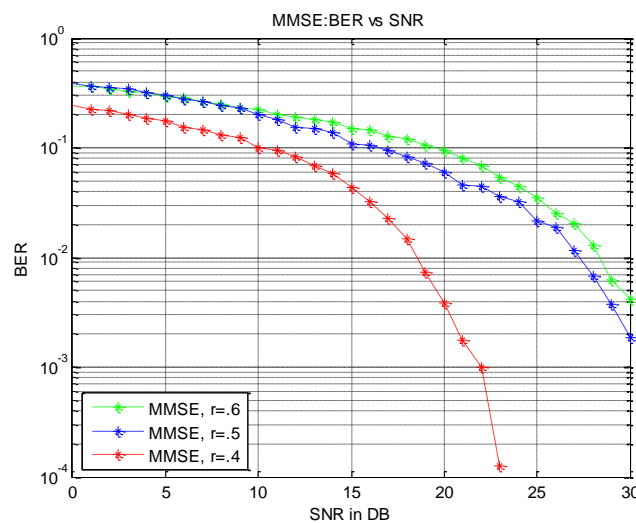


Figure 8 BER comparison MMSE with respect to correlation coefficients

The parameter consider for simulation is (number of transmitter) $T_x=2$ and (number of receiver) $R_x=2$, .5 correlation coefficients, 4, 8, 16 and 32 bits training sequence and QPSK modulation technique for data transmission. Figure 11 and 12 show BER curves for 2x2 MIMO systems using flat fading channel by MMSE and Ls estimation with respect to different bit training values which show that BER decreases with large bit training values.

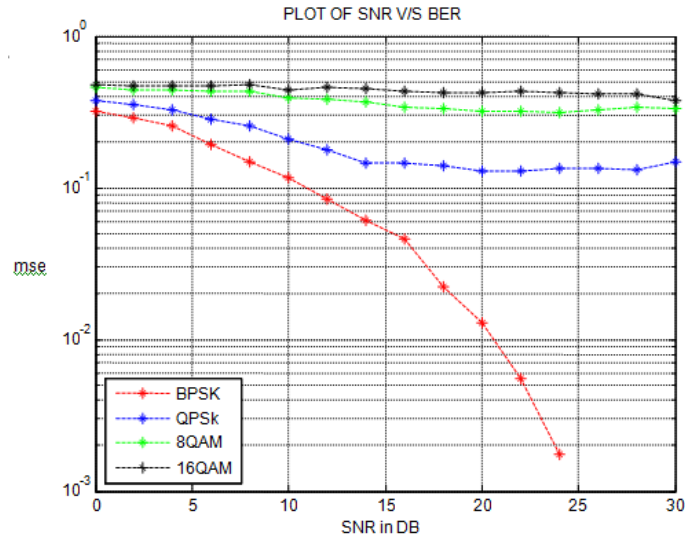


Figure 9 BER comparison LS with respect to different modulation techniques

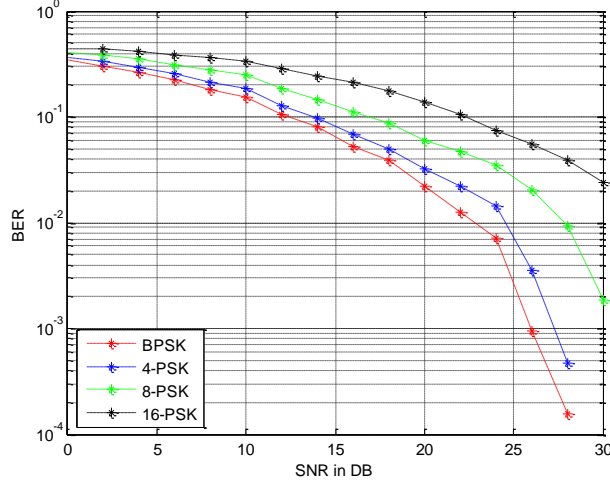


Figure 10 BER comparisons MMSE with respect to different modulation techniques

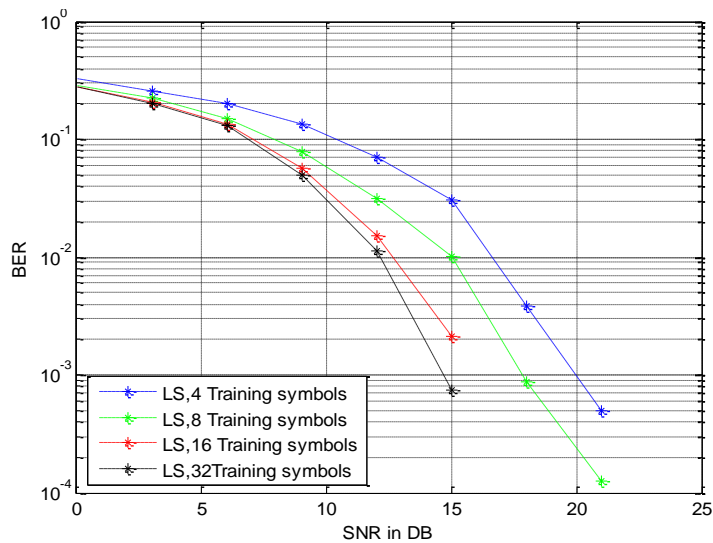


Figure 11 BER comparison LS with respect different bit training values

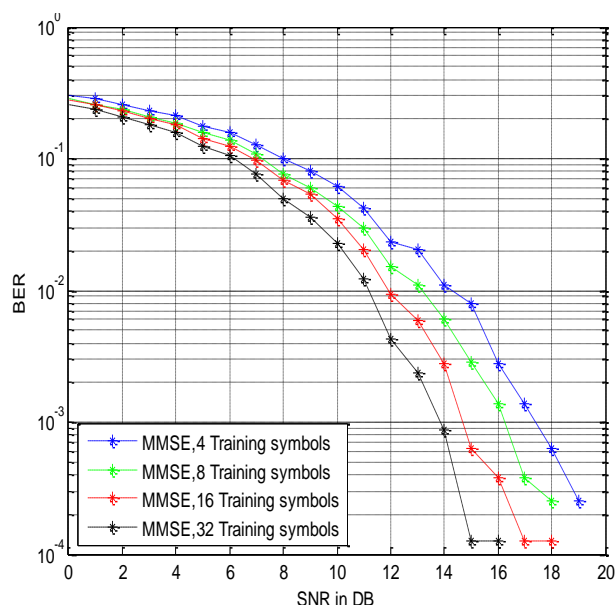


Figure 12 BER comparison MMSE with respect different bit training values

V. Conclusion

MIMO systems play a vital role in fourth generation wireless systems to provide advanced data rate. In order to attain the advantages of MIMO systems, it is necessary that the receiver and/or transmitter have access CSI. The time-varying nature of the channel typically requires the use of frequent channel retraining, which in turn increases the data overhead due to training signals, thus reducing the system's overall spectral efficiency. Hence, effective channel estimation algorithms are needed to guarantee the performance of communication. In this project, a number of channel estimation algorithms have been implemented and evaluated. In this chapter, training based channel estimation schemes in flat fading MIMO systems are investigated. After introducing LS and MMSE estimators, they are simulated in a flat fading MIMO channel considering AWGN.

Simulation results show that The algorithm of LS estimator is very simple, As LS algorithm does not require correlation function calculation nor does it require matrix inversion. MMSE estimator is complex; As MMSE algorithm requires both correlation function calculation and matrix inversion. From the Simulation results it is clear that MMSE estimator provides better performance than LS estimator in terms of mean square error (MSE) and Bit error rate (BER) whereas implementation of LS algorithm is much easier than MMSE algorithm Also Simulation results show that the BER for 2x2 MIMO systems using flat fading channel with different correlation values decreases with low correlation values. The results demonstrate that SNR required to support different values of bit error rate varies depending on different low correlation between the transmitting and the receiving antennas. In addition, it is illustrated that when the number of and receiver antennas increases, the performance of TBCE schemes significantly improves. Also The results show BER curves for 2x2 MIMO systems using flat fading channel by MMSE and Ls estimation with different bit training values which show that BER decreases with large bit training values. As from the general modulation theory we know the performance is better for less order modulation technique as compared to high order modulation but in high order modulation we have larger data rate. Same behavior is also observed for MIMO system. Performance both MMSE and LS estimation are same for all kinds of modulation at small value of SNR but as we increase the SNR value the performance gap goes on increasing.

References

- [1]. Joseph, W. Reynders, W. Debruyne, J. and Martens, L., "Influence of Channel Models and MIMO on the Performance of a System based on IEEE 802.16", Wireless Communications and Networking Conference, ISBN 1-4244-0659-5, pp.1826-1830, 11-15 March, 2007.
- [2]. George Tsoulos, "MIMO System Technology For Wireless Communications", Revised Edition, CRC Publisher, 2006.
- [3]. Yan tao Qiao, Songyu Yu, Pengcheng Su and Lijun Zhang, "Research on an Iterative Algorithm of LS Channel Estimation in MIMO OFDM Systems", IEEE Transactions on Broadcasting, vol. 51, no. 1, pp. (149-153), March 2005.
- [4]. BPSK/QPSK Modulation and Demodulation. Retrieved February 11, 2010, from Free Online Course Materials — USU Open Course Ware.
- [5]. C.R. Murthy, A.K. Jagannatham, and B.D. Rao, "Training-Based and Semi-Blind Channel Estimation for MIMO Systems with Maximum Ratio Transmission", IEEE Transactions on Signal Processing, Vol. 54, No. 7, 2006.
- [6]. M. Pukkila, "Channel Estimation Modeling", Postgraduate Course in Radio-Communications, Nokia Research Center, Fall 2000.
- [7]. Tianbin Wo, Peter Adam Hoeher, Ansgar Scherb, Karl-Dirk Kammeyer, "Analysis of Semiblind Channel Estimation for FIR-MIMO Systems", German Research Foundation (DFG), October 2005.

- [8]. XiaoyunHou, BaoyaXeng, HanwenLuo, Wenato Song, Hong-Bin, "Channel Estimation for MIMO- OFDM based Wireless Networks", IEEE Vehicular Technology Conference, VTC, Vol. 4, pp.1883-1887.
- [9]. Chia-Liang Lui, "Impacts of I/Q Imbalance on QPSK –OFDM-QAM Detection", IEEE Transactions on Consumer Electronics, vol. 44, no. 3, pp. (984-989), August, 1998.
- [10]. A.J. Paulraj, R.U. Nabar and D.A. Gore, "Introduction to Space Time Wireless Communication, Cambridge, UK: Cambridge University Press, 2003.Mertins, Signal Analysis. Wiley,1999.
- [11]. Zelst and J.S. Hammerschmidt. 2002. "A single coefficient spatial correlation
- [12]. models for multiple-input multiple output (MIMO) radio channels," in Proc.URSIXXVIIth General Assembly, 2002.
- [13]. Comparison of different models for the analysis of Rayleigh fading channels." -- PDF article.
- [14]. D.S. Shiu, G. Foschini, M. Gans and J. Kahn, "Fading correlation and effect on the capacity of multielement antenna systems", IEEE Trans. Commun., vol. 48, no. 3, March 2000, pp. 502–512.
- [15]. D. Chizhik, G. Foschini, M. Gans and R. Valenzuela, "Keyholes, correlations and capacities of multielement transmit and receive antennas", Proc. Vehicular TechnologyConf., VTC'2001, May 2001, Rhodes, Greece.